

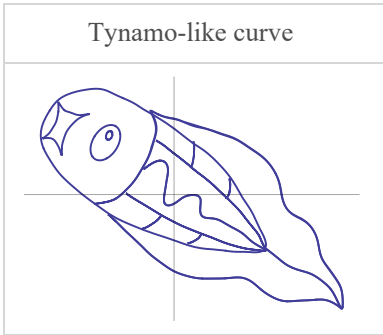
# MATH:1260 Pokémath

## The Mathematics of Pokémon Go<sup>®</sup>

Week 8 Wednesday, Spring 24

Popular curve:

Tynamo-like curve



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## Plan for Today

- Module 2: Gotta Catch 'Em All!®
  - Conditional probabilities
  - Probability of catch

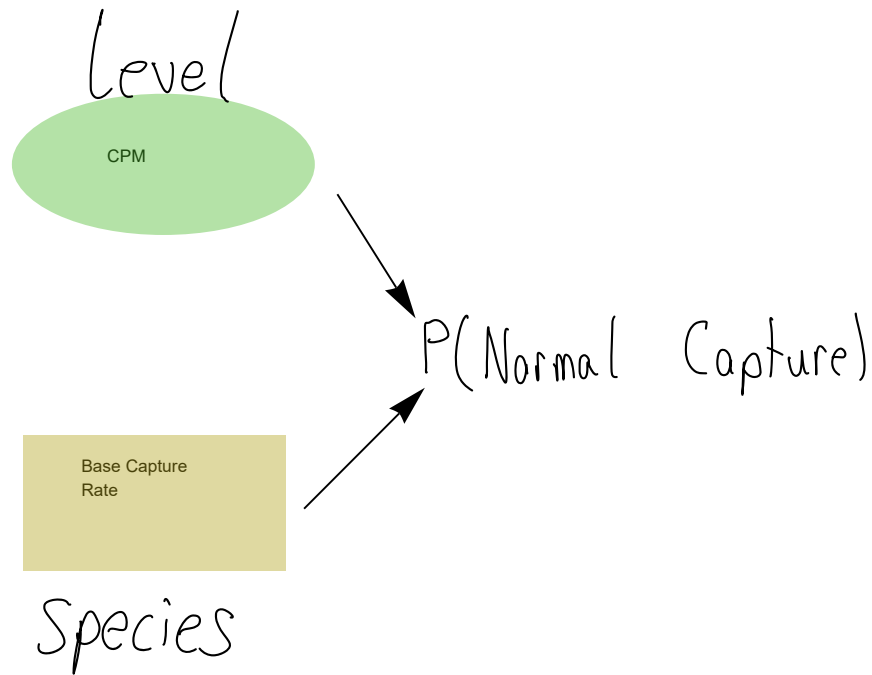
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## Class Reminders

- GW7 in discussion Thursday.
- HW6 due Wednesday after break.
  - We will have a partial work day for the assignment on Monday
- Project 2 Stage 1: Proposal is ALSO due Wednesday after break.
  - Investigating Spawn Rates!
- No class next week.

## Catch Rates


**Normal Capture:** One regular ball, no curve, no nice throw, no berry, no boost.



$$P(\text{Normal Capture}) = \frac{\text{Base Capture Rate}}{2 * \text{CPM}}$$

Tynamo!

My Tynamo is level 13. What is the P(Normal Capture)?

			Base Capture Rate
602		Tynamo	40%
level	CPM		
12	0.4627984	1300	
12.5	0.472336093	1300	
<u>13</u>	<u>0.48168495</u>	1600	
13.5	0.4908558003	1600	
14	0.49985844	1600	

$$P(\text{Normal Capture}) = \frac{4}{2 \cdot 0.48168495}$$

$$= 0.415$$

## P depends on Level

Let's compare a level 13 Tynamo to a level 30 Tynamo

Level 13 Tynamo has a Prob of Normal Capture of about .42

Level 30 Tynamo has a CPM of .7317. This gives a Prob of Normal Capture of about .27

But those probabilities only matter if the wild Tynamo I find is level 13 or level 30. So there is a random element to the level as well..

2 Probabilities:  
Tynamo's Level  
P(Normal Capture) } dependent events

## Conditional Probability

When we have a piece of information that can change the probability of an event in our random experiment, we need to think about conditional probability!

Some notation:

$P(A|B)$  represents the probability of A **given** B. That is, what is the probability of event A if we **already know** event B has happened.

L13 is the event Tynamo is level 13

L30 is the event Tynamo is level 30

C is the event Tynamo is captured

$$P(C|L13) = .42$$

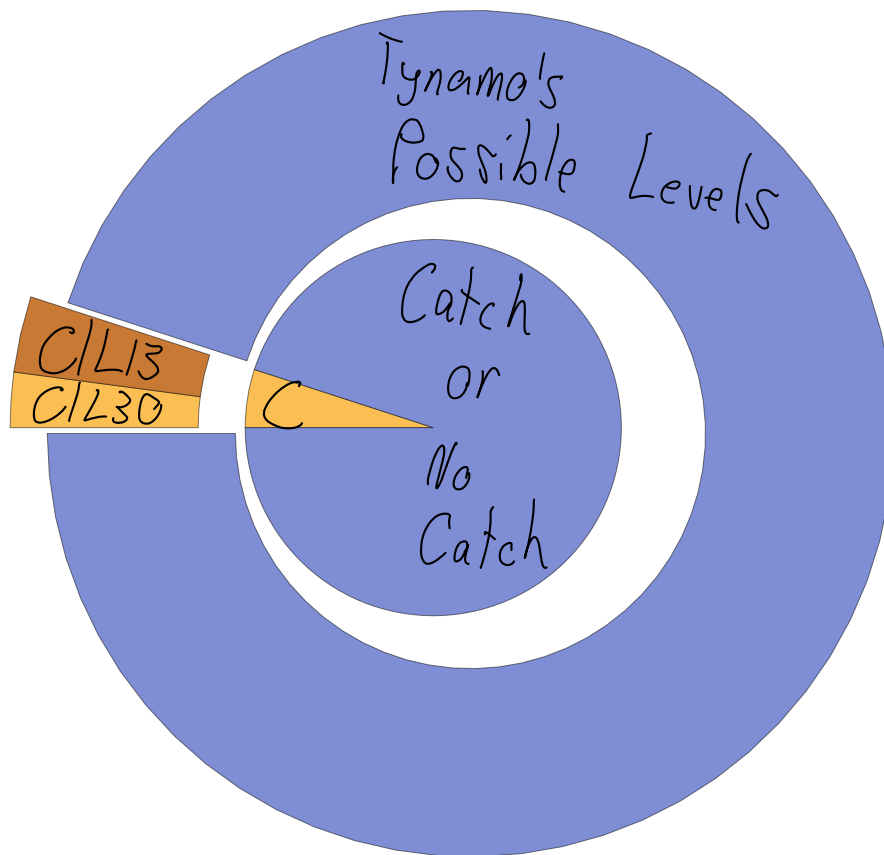
$$P(C|L30) = .27$$

$$\left. \begin{array}{l} P(L13) = \frac{1}{30} \\ P(L30) = \frac{1}{30} \end{array} \right\} \text{for a level 30} \\ \text{trainer}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

## Thinking about proportions

This example



## General Multiplication Rule of Probability

$$P(A \cap B) = P(A | B) * P(B)$$

This is the intersection rule we couldn't do before! With conditional probability, we have the tools.

Note: A, B need not be independent! We only need the probability of A **given** B.

$$\begin{aligned} P(C \cap L13) &= P(C | L13) \cdot P(L13) \\ &= .42 \cdot \frac{1}{30} = .014 \end{aligned}$$

Tophat: What is  $P(C \cap L30)$ ? In words: what is the probability of catching a Tynamo and the Tynamo is level 30?

$$\begin{aligned} P(C \cap L30) &= P(C | L30) \cdot P(L30) \\ &= .27 \cdot \frac{1}{30} = .009 \\ &\quad .01 \end{aligned}$$



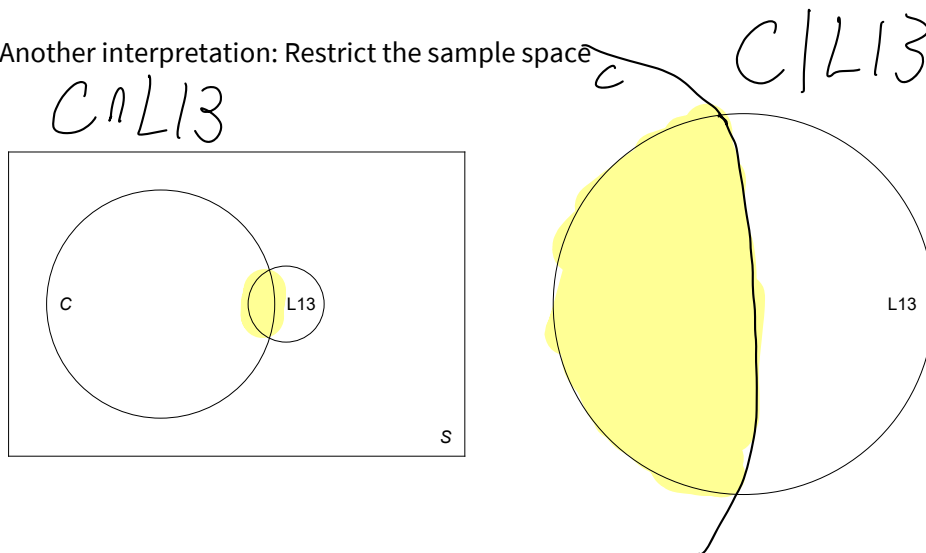
This can be written another way!

The conditional probability of A given B is

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

**This is the same as the previous formula, just in a different arrangement.**

Another interpretation: Restrict the sample space



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## Formulas from last time:

### **Probability of a Pokemon escaping “n” times in a row:**

$$P(n \text{ escapes in a row}) = (1 - P(\text{Normal Capture}))^n$$

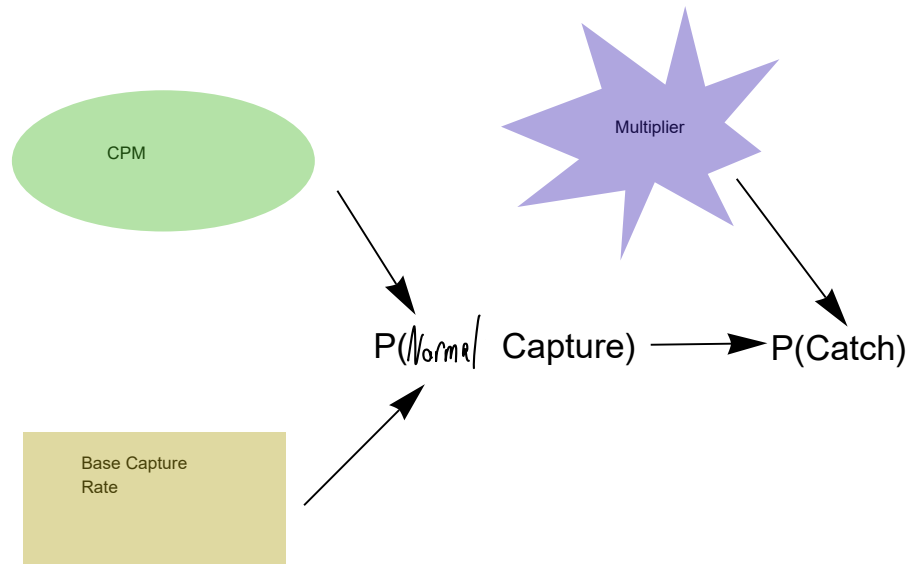
This one comes from using the product rule for independent events “n” times in a row.

### **Probability of capturing a Pokemon within “n” Pokeballs**

$$P(\text{catch within “n” balls}) = 1 - P(n \text{ escapes in a row}) = 1 - (1 - P(\text{Normal Capture}))^n$$

This one comes from the fact that if a Pokemon did NOT escape “n” times in a row, it must have been caught at least once in “n” attempts.

## Beyond Basic Capture



Here is the upgraded formula from Normal Capture to a proper **Catch Probability**.

$$P(\text{Catch}) = 1 - (1 - P(\text{Normal Capture}))^{\text{multiplier}}$$





This looks similar to the previous formula by design. However, this one was made up by the developers of Pokemon GO.

## How to Compute Multiplier

Multiplier = Ball \* Berry \* Throw \* Curveball \* Encounter \* Medal

[https://bulbapedia.bulbagarden.net/wiki/Catch\\_rate\\_\(GO\)](https://bulbapedia.bulbagarden.net/wiki/Catch_rate_(GO))

### ■ Ball is

- 1 if a  Poké Ball or  Premier Ball is thrown
- 1.5 if a  Great Ball is thrown
- 2 if an  Ultra Ball is thrown

↳  $P(\text{Catch})$

$$= 1 - (1 - P(\text{Normal Capture}))^2$$




→ Pokemon effectively has to escape 2 Normal Captures in a row to escape a single Ultra Ball.

### Example





Suppose instead that I use an ultra ball (no other “boosts” in my multiplier). What is the probability I catch Tynamo?  $Lvl\ 13$

$$1 - (1 - .42)^2 \approx .663$$

TH What if I throw a great ball and use a golden raspberry?

- **Berry** is
  - 1.5 if a  **Razz Berry** is used
  - 1.8 if a  **Silver Pinap Berry** is used
  - 2.5 if a  **Golden Razz Berry** is used
  - 1 if otherwise

- Throw is  $2 - r$  if the Ball hits within the target ring, and 1 otherwise
  - where  $r = (\text{target ring size}) / (\text{maximum ring size})$ , hence  $0 < r \leq 1$
  - for a Nice Throw,  $1 \leq \text{Throw} < 1.3$
  - for a Great Throw,  $1.3 \leq \text{Throw} < 1.7$
  - for an Excellent Throw,  $1.7 \leq \text{Throw} < 2$

- `Curveball` is 1.7 if the Ball is spun before being thrown, and 1 otherwise
  
- `Encounter` is 2 if the Pokémon is encountered for completing [Field](#), [Special](#) or [Timed](#) Research tasks, and 1 otherwise
  
- `Medal` is based on the player's [type-specific Medals](#) pertaining to the wild Pokémon's type and is
  - 1 if  none
  - 1.1 if  bronze
  - 1.2 if  silver
  - 1.3 if  gold
  - 1.4 if [20px](#) platinum
  - if the wild Pokémon has two types, `Medal` will be the average of the above for each type